Introducing the wave length associated with the frequency ω

$$\lambda = 2\pi c/\omega$$

we obtain

$$\mathscr{W} = 16\pi^4 e^2 x_0^2 c/3\lambda^4 \tag{A-7a}$$

According to this result, we see that if a group of oscillators have the same amplitude, x_0, but different frequencies, they will emit radiation in inverse proportion to the fourth power of the wave length of the radiation. The reason for this high power is that for harmonic motion the acceleration is proportional to the square of the frequency, and the intensity of the radiation is proportional to the square of the acceleration. We shall see that this simple fact explains the blue color of the sky (page 586).

Assumption 4b. It is possible to show from Maxwell's equations that a magnetic dipole whose moment varies sinusoidally with the time will emit radiation in which the direction of the magnetic field is in the same plane as the direction of the dipole. The field strengths in this radiation have exactly the same magnitudes as those from an oscillating electric dipole of equal amplitude. That is, the amplitude m_0 in (A-5b) can represent either a magnetic or an electric dipole moment, the units of measurement being respectively e.m.u. and e.s.u.

Thus we can have electric dipole radiation and magnetic dipole radiation. Later in this chapter we shall also consider a third type of radiation—that produced by quadrupoles. Of these three types, however, the electric dipole radiation is usually the most important by far. In most of the discussion in this and the next chapter, therefore, the term "dipole" refers to an electric dipole, and the effects of quadrupoles will not be considered.

The four assumptions discussed above are perfectly general and are not restricted to the electron theory of matter. The properties of the electron are introduced in the following assumption.

Assumption 5. The electron is a particle that moves according to the laws of Newtonian mechanics. Its motion is therefore determined by the forces acting upon it. These forces consist of the following:

(*i*) An inertial force, $\boldsymbol{f}_1 = -m_e\boldsymbol{a}$, where m_e is the mass of the electron and $\boldsymbol{a}$ is its acceleration.

(*ii*) The restoring force responsible for the harmonic motion the electron is assumed to undergo in the atom, $\boldsymbol{f}_2 = -k\boldsymbol{x}$, where $\boldsymbol{x}$ is the displacement of the electron from its equilibrium position and k is a constant. The origin of this force was never made clear in the classical electron theory.

(*iii*) The force on the electron due to whatever electric fields are present, $\mathbf{f}_3 = e\mathbf{E}$, where e is the charge on the electron.

(*iv*) The force on the electron due to any magnetic field that might be present, $\mathbf{f}_4 = e(\mathbf{v} \times \mathbf{H})/c$, where $\mathbf{v}$ and $\mathbf{H}$ are the velocity and magnetic field strength (both vectors) and $(\mathbf{v} \times \mathbf{H})$ represents the vector product (page 26). In a light wave, since $|\mathbf{E}| = |\mathbf{H}|$, this force will be negligible compared with $\mathbf{f}_3$ as long as the velocity of the electron is small compared with the velocity of light ($|\mathbf{v}| \ll c$). This is always true in chemical systems. We therefore conclude that when light interacts with ordinary matter, it is generally the electric field of the wave that does most of the interacting, rather than the magnetic field. There are, however, some exceptions (e.g., in the phenomenon of optical rotation, where the interactions with both magnetic and electric fields in the light wave are important).

(*v*) Since a moving electron may dissipate energy in the form of radiation, there must be a force associated with the emission of radiation. This is called the *radiation damping force*. It is convenient for later applications to assume that this force is proportional to the velocity, $\mathbf{f}_5 = -\mu\mathbf{v}$. The constant μ is called the radiation damping coefficient, and its value can readily be calculated for an electron undergoing harmonic motion. The rate of energy dissipation as radiation is $-(\mathbf{f}_5 \cdot \mathbf{v}) = \mu|\mathbf{v}|^2$. The energy radiated per second by the oscillating electron is (taking $\mathbf{x} = \mathbf{x}_0 \cos\omega t$ and $\mathbf{v} = d\mathbf{x}/dt$)

$$\mathscr{W} = \text{(number of cycles per second)} \times \text{(energy radiated per cycle)}$$

$$= \nu \int_0^{1/\nu} |\mathbf{f}_5||\mathbf{v}|\, dt = \nu\mu x_0^2\omega^2 \int_0^{1/\nu} \sin^2\omega t\, dt$$

$$= \nu\mu x_0^2\omega \int_0^{2\pi} \sin^2 x\, dx = \mu x_0^2\,\omega^2/2$$

But according to Equation (A-7)

$$\mathscr{W} = e^2x_0^2\omega^4/3c^3$$

Thus the radiation damping coefficient is

$$\mu = 2e^2\omega^2/3c^3 \qquad \text{(A-8)}$$

(*vi*) It is also assumed that the electron can dissipate energy in forms other than light—particularly as heat. The forces responsible for this are called frictional forces, or "damping forces" because they tend to "damp out" the